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Developing 3D Box Counting Approach for Built Form Visual Assessment

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Abstract

This research aims to develop a computational approach for visual assessment of built form reducing interventional subjectivity. Such an approach is supposed to serve as an accurate practical tool of built form assessment for different purposes, such as legal action and identity preservation. Fractal geometry has dominated the field of visual assessment recently as a computational tool; fractal dimension is the mathematical representation of visual complexity of architecture. Only two-dimensional box counting approaches have been applied and tested in both manual and computerized applications. In this paper, a 3D box counting approach has been developed and manually applied on Robie house, to calculate fractal dimension, comparing it to the previous two-dimensional results for the same building. The results found mathematically and architecturally valid. The developed approach needs to computerization in future for more accuracy.

Keywords:	Built form; Computational visual assessment; Fractal dimension; Box counting

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1. Introduction

Quantitative visual assessment approaches acquired increasing importance in the fields of town planning legislations, heritage preservation and architectural design. Several tools and techniques emerged through the last three decades. It appears that fractal geometry as a computational tool has dominated the field of quantitative assessment recently. Fractal geometry has been widely used to calculate the fractal dimension for 2D facades, which is the mathematical representation of visual complexity.

Fractal dimension is a quantitative technique of visual analysis that has been widely used in different fields including architecture; Bovill, C. used it as a tool to calculate visual complexity of buildings' façades via two-dimensional images [2]. The calculation of fractal dimension can be performed using the box counting method. Tucker, C. [11] used an automated technique developed from the work of Bovill, C. [2], using a methods developed by Foroutan-pour, K. [3] to determine the best approximation for the fractal dimension. Ostwald and Vaughan fine-tuned and refined the 2D box counting approach, reducing its errors, through the application of comparative analysis [12] retesting Bovill's method [13] and optimizing pre-processing standards [9].

2. Objectives

The objectives of this research are about developing the visual domain in architecture by investigating built form, through the development of a method of visual investigation, which can be used in both practice and research.

The major objective of this research is to generate an approach to investigate the visual appearance of built form, **reducing interventional subjectivity** within architectural visual judgment. This approach should achieve the following:

- To employ computational tools that based on the nature of architectural form, in order to generate quantitative representation of visual quality.
- To enable visual comparison benchmarking by being mathematically and architecturally valid.
- To provide a guide for building and **planning legislations**, this means that it has to be practical and accurate.
- To involve **digital tools** in visual form assessment of architecture by making the approach programmable as an algorithm.

3. Fractal geometry

Fractal geometry is a term founded in the late 1970's by Mandelbrot, B. [6]. He stated that "*a fractal is a rough fragmented geometric figure that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole; and it has two properties: self-similarity and fractal dimensions*" [6] or in other words, their topological dimension is not an integer. He coined the term fractal from the Latin adjective "fractus". Fractals provided a mathematical model for many complex objects found in nature such as coastlines, mountains, and clouds. Mandelbrot, B. [6], discussed that Euclidian geometry, is unable to describe nature complexity, because mountains are not conical, clouds are not spherical, and rivers are not orthogonal; these objects are too complex to possess characteristic sizes and to be described by traditional Euclidean geometry [4].

3.1. Fractal visual analysis of architecture

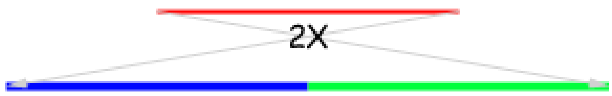
According to Ostwald, M. [8], fractal analysis is one of the most widely used techniques for quantitative visual assessment for the built environment character. Many scholars have applied fractal geometry to analyze non-linear urban forms at the regional, city, and building levels [5]. According to Tucker, C. [10], architectural designers adopted fractal geometry shortly after its emergence by Mandelbrot, B. [6]. Later it has been involved in built environment visual analysis by many authors. According to Ostwald, M. [8], "*since 1990s, fractal geometry has been used to measure the properties of town plans and skylines*" [8]. Architectural researchers have also used manual fractal analysis with different manipulations, to measure the visual properties and visual complexity of contemporary architecture and historic buildings.

3.2. Fractal dimension

The idea of fractal dimension emerged when Mandelbrot, B. [7], presented the measured dimension through a discussion of the length of the coast of England. According to Li, J. [4], Fractal dimension (FD) is a useful feature for texture segmentation, shape classification, and graphic analysis in many fields. The box-counting approach is one of the frequently used techniques to estimate the fractal dimension of an image. A fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a fractal pattern changes with the scale at which it is measured.

3.3. Concept of magnification

Geometrical topological dimension represents how many dimensions the figure contains, but mathematically, a one dimensional object if magnified 2 times, will result 2 times the original (Figure1-a). In addition, a two dimensional object magnified by two will result 4 times the original (Figure1-b), and a three dimensional object magnified 2 times will result 8 times the original (Figure1-c). The magnification is constant, but the result changes in accordance with the topological dimension. This means that the topological



Figrue 1-a. Dimensional line segment with a magnification of 2 [15]

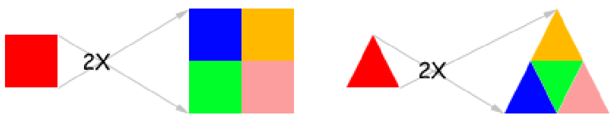


Figure 1-b. Dimensional square and triangle with magnification of 4 [15]

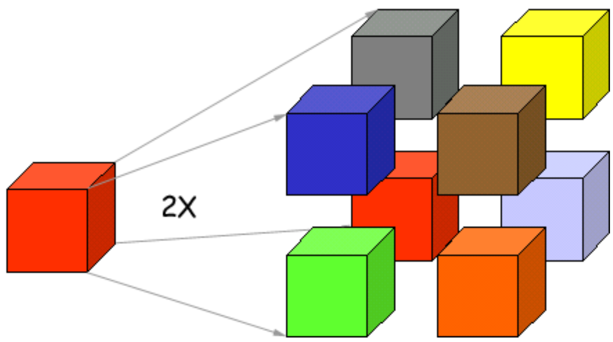


Figure 1-c. Dimensional cube with the magnification of 2 [15]

dimension also represents how a figure would scale up. The mathematical analysis shows form the previous values that the resulting shape multiplications is equal to the magnification factor raised to the power of the dimension value.

From the previous examples, it is clear that the mathematical relationship between "D" as the dimension, "N" as the number of multiplications, and "e" as the magnification factor, is according to the following equation [14]:

$$e^D = N \quad (1)$$

Solving the equation for D, the formula becomes [14]:

$$D = \log N / \text{Log } e \quad (2)$$

But applying the equation for a fractal shape such as the Kock curve will result in a fractional dimension. Because with a magnification factor 3, the resulting multiplications of the details are 4. Applying the equation:

$$D = \log 4 / \log 3 = \mathbf{1.26} \quad (3)$$

This is why they call it a fractal dimension. This equation is known as Hausdorff-Besicovitch dimension equation to calculate the self-similarity dimension D_s [14].

3.4. Bovill's method to calculate fractal dimension for architecture

Bovill, C. [2] performed box counting method for calculating fractal dimension, to determine the approximate visual complexity of architecture. According to Vaughan, J. [12], Bovill's method is one of only a limited number of quantifiable approaches to provide a measure of the relative complexity of architectural forms. The approach of Bovill, C. [2] was based on his assumption that architecture is necessarily produced through the manipulation of rhythmic forms. He suggested that fractal geometry can be used to measure order and surprise.

Bovill's box size: Computational fractal dimension calculation methods use a range of scaling coefficients to gradually reduce the grid size and generate more accurate result. Vaughan, J. [13] discussed the determination of box size by Bovill, C. [2]. She stated that Bovill considered the optical action of the eye focusing from different distances, assuming the location of the viewer between 40-80 feet away. He then calculated the approximate grid size for the different scales, based on the distance of observation and the angle of the field of view, or the size of the focusing point.

Bovill's process: The process of calculating the Box-counting dimension of the Robie house was performed in the following steps:

Superimpose a grid of square boxes over the image (grid size s_1)

Count the number of boxes that contains some of the image ($N_{(s1)}$)

Repeat this procedure with smaller and smaller grid size ($s_{2,3,\text{and } 4}$)

Then apply the equation:

$$D_b = \frac{[\log (N(s2)) - \log (N(s1))]}{[\log (s_2) - \log (s_1)]} \quad (4)$$

For the Robie house, Figure 2 illustrates the procedure of box-counting as performed by Bovill, [2]. The fractal dimension of the Robie house elevation is about 1.44 or a bit more.

4. Title

The third dimension needs to be involved to complete the visual field and to generate results that can be valid as a simulation of our three dimensional world. The human eye can receive and interpret a three dimensional image. If the visual assessment approach is about measuring the visual impact of the building, and to simulate the perceptual algorithms of the human eye; then the third dimension has to be involved. Because a two-dimensional visual assessment may not generate perfect results that can represent our visual interaction. To involve the third dimension, a complete three dimensional visual assessment is the best way, that can involve the entire physical entity of the built form, and it can generate a result which is valid from all points of view, and represents the real image of the built form regardless the differences of viewers.

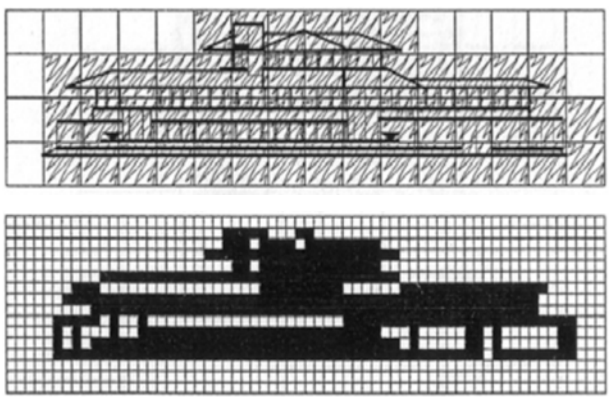


Figure 2. Box-counting dimension of the Robie house [2]

Most of the problematic parameters within the box counting methods are because it is two-dimensional. Such as the lines representation and details levels, which will be eliminated in a three-dimensional approach. Because a real physical volume will be used. In addition, depth, recesses, projection and horizontal layout will be involved because a full three-dimensional model will be used. It is mathematically possible to calculate the fractal dimension using a 3D box counting process for a 3D object, although it does not have a practical application yet. The same concept of 2-dimensional plains is applicable on three-dimensional objects. A 3D box counting is mathematically possible, using the same equation as in the 2D box counting.

4.1. Technical difficulties within 3D box counting approach

A key problem for the box-counting method is that many different scale grid comparisons are needed to get a useful or accurate answer. Ideally, at least 13 different scale grids are needed. Bovill, C. [2] does just 2 or 3 grid comparisons, and that's why his results are highly inaccurate. Some researchers argued that over 25 different scale grid comparisons are required. This is difficult enough for 2D, but for 3D, it needs a lot of computing power.

The difficulty in such an approach is that generating 3D cubes with the same size over the model will cover the entire model and make invisible even for a computer, unless the cubes were transparent and different in its digital nature to be differentiated by the computerized algorithm or by the bare eyes in a half manual application. In this research, only a half-manual application is about to be performed. Which means that the building has to be modelled as one object. Then boxes intersects with its skin or located inside it (in case the building was considered as a solid volume) has to be detected and counted. On a computer screen, this might be problematic, unless a working algorithm or a system of work is generated and followed.

4.2. 3D box counting approach method

To perform a three-dimensional box counting, there is a need to generate an algorithm. It should be able to generate and analyse boxes through a systematic procedure; then to count the boxes that partially or totally located within the volume of the building. The process should be taken in several iterations to calculate the box-counting dimension, using the same formulae of 2D box counting. This fact is mathematically based on the one-dimensional denominator for all topological dimensions.

$$D_b = \frac{[\log (N(s_2)) - \log (N(s_1))]}{[\log (s_{2)} - \log (s_{1)}]} \quad (5)$$

There are two options available to work on a 3D model, the first option is to consider the building as a solid volume, and the second option is to consider the building as an external skin only, which means to count only the boxes intersect with the external skin. The second option is to consider the building as a solid volume, and in that case, all the boxes within the building space have to be counted. It has been found that it is mathematically valid and more practical to consider the building as a solid volume; because the external skin thickness will not be mathematically accurate, and its invisible depth is not architecturally perceivable.

The white space should be determined according to Ostwald, M. [9], which according to his assessment a 50% larger than the object. In a 3D model, the white space should be increased by 50% in x, y and z-axis. Using the same white space will generate comparative benchmark to validate the 3D box counting approach based on the validity of the 2D box counting approach.

The cube deletion process

The deletion process is suggested as a series of steps on an algorithm for the 3D box counting. This process is the easiest way to accurately eliminate the boxes that are not interfering with the building volume, considering the building as a solid entity. This process depends on the deletion of every box once on the row, and if the building volume appeared on the deleted box space, the deletion process will be undone, and if the volume of the building didn't show up on the deleted box space, the deleted box will be permanently removed.

In case of manual box counting, the boxes should be non-transparent, and they should have a contrast colour, because the process will depend on direct vision. This process would be performed in order, starting with one corner, until the top layer is finished, then to the next layer, and so on. On a manual work, it will be a challenge to remember which boxes have been processed and which are not, so an accurate order has to be used. It is suggested to work on the top first until the top of the building is reached, then to work on each side, until the façade of the building is reached, then another round of all surface boxes has to be made to make sure that all the outside boxes has been removed. Then another test has to be done to assure that all the boxes have been processed, which is to turn the model to wire frame, to look inside and to check from all sides that there are not a single box free from the building and not deleted. The final step is to select the remaining boxes by name or by type and command the software to count them. The number of the remaining boxes is (N), and the number of the boxes at the bottom layer of the longest side (before deletion) is (e) or (1/s). This process should be repeated for several scales, then the following equation has to be -applied for each two scales results:

$$D_b = \frac{[\log (N(s2)) - \log (N(s1))]}{[\log (s_{2)} - \log (s_{1)}]} \quad (6)$$

5. Experiment on Robie house for validation

This is an attempt to perform 3 dimensional box counting on buildings 3D models, using Autodesk 3ds Max® program as a three dimensional world. A 3D model of the Robie house has been chosen to perform a test application for this approach using almost the same grid sizes of Bovill but as three-dimensional cubes, to be able to validate the results using the benchmark of the previous studies about the Robie house. This experiment was performed manually and the results were between 2.54 and 2.65, which seem to be not only mathematically valid, but also conform to the results of Bovill, [2] and Vaughan, J. [13]. The same formula of the 2D box counting has been used, according to the mathematical background of the three-dimensional box counting formula. The building has been considered as a solid volume, and the box counting was performed with the cube deletion process.

Iteration 1

The first step is to determine the size of the external box that should contain the grid around the building. The same Bovill's external box was used, which is 60x24x12 meters, which matches the grid size in the two dimensional box counting of the two elevations of the Robie house, performed by Bovill, [2]. The grid has been divided to 3D cubes with 6 meters size, compared with the 7.2 meters grid size of Bovill, but the 6 meters has been chosen to have simple number without fractions, see Figure 3.

In a shaded view is used to start the cube deletion process, as in the following figure, any cube that was deleted and the building components appears in its position, will be undeleted, until all the empty boxes are removed (Figure 4a and b).

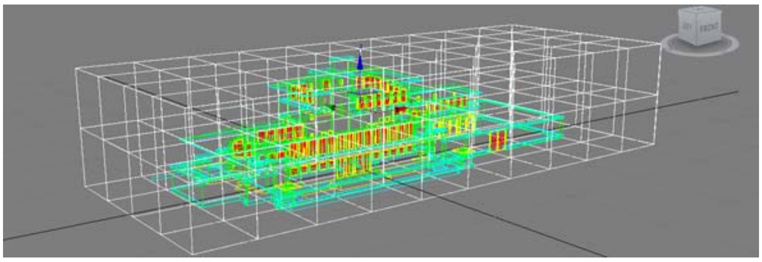


Figure 3. Robie house Iteration 1 in 3D (wire frame)

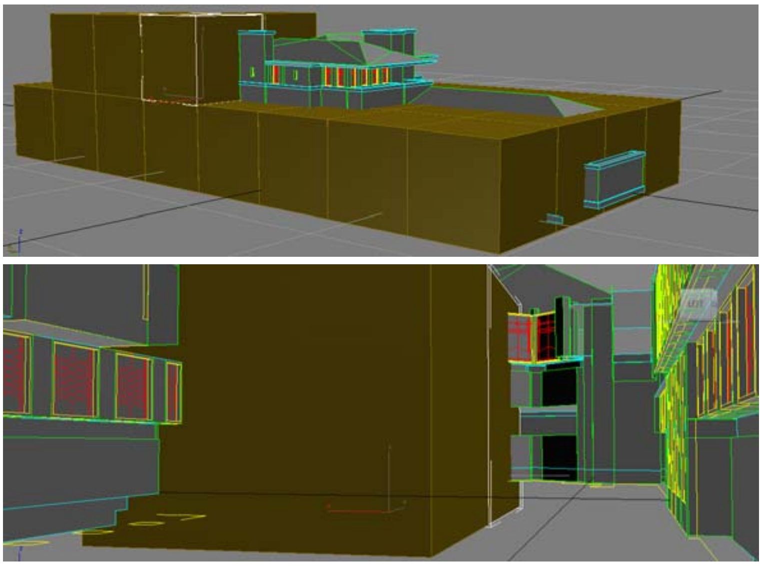


Figure 4. Cube deletion process for Iteration 1

a) Top: Deleting the boxes and retrieving any box that reveals the building
b) Bottom: The boxes inside the volume of the building are also counted

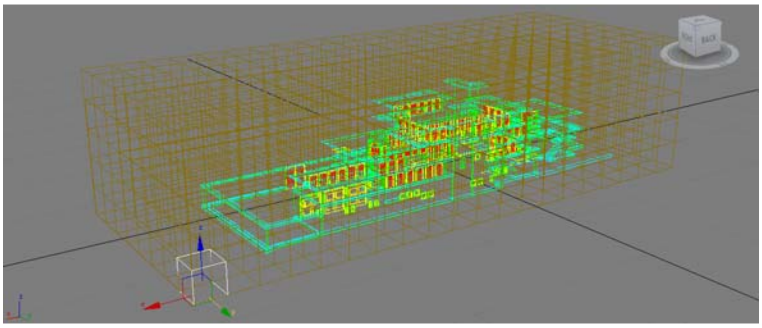
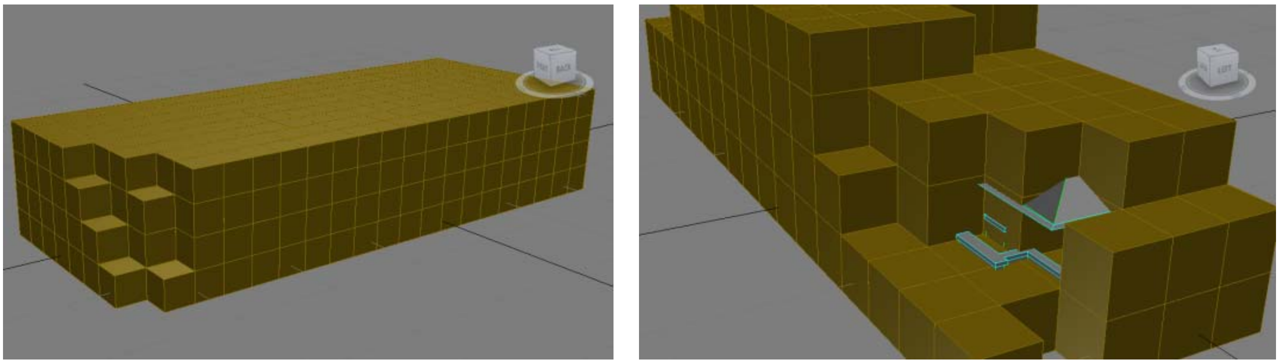


Figure 5. Robie house Iteration 2



a) Deleting the boxes b) finishing the deletion process

Figure 6. Robie house cube deletion process for Iteration 2

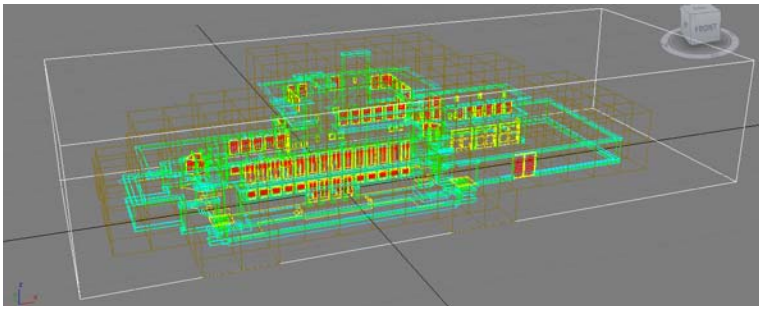
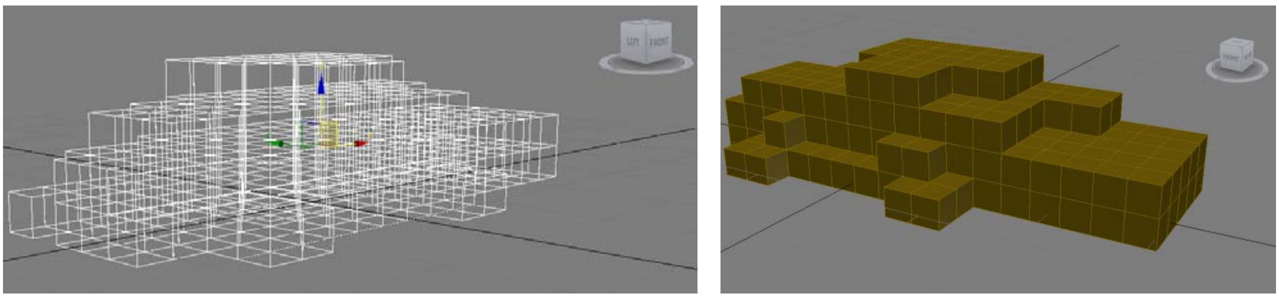
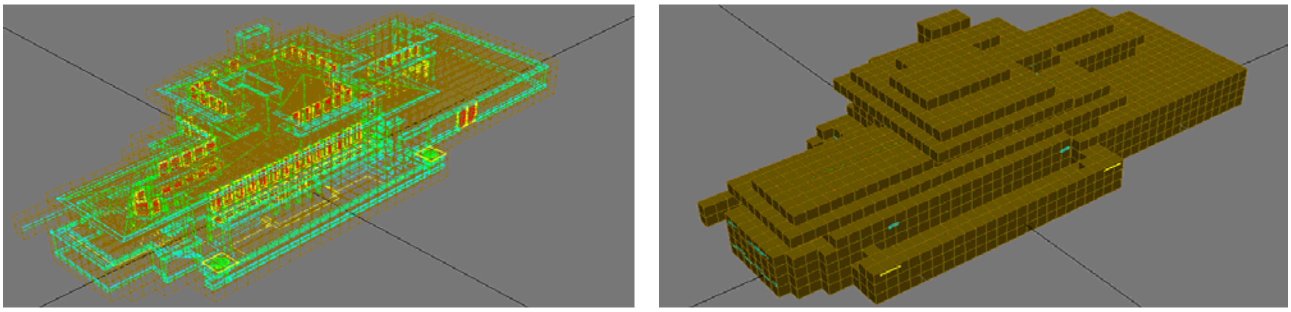


Figure 7. Iteration 2 wire frame check



a) Boxes wire frame b) Boxes forming the shape of the building

Figure 8. Forming the shape of the building



a) Iteration 3 box formation wire frame model b) Iteration 3 box formation shaded model

Figure 9. The box formation is similar to the original form

Table 1: Robie house 3D box counting

Iteration	Total boxes	Box size	Scale factor (S)	Log S	Count (N)	Log N	D(self-semi.)
1	80	6	10	1	50	1.96	1.69
2	640	3	20	1.3	257	2.4	1.84
3	17280	1	60	1.77	4567	3.65	2.06

Iteration 2

The second iteration, the box size became 3 meters (Figure 5).

The boxes in iteration 2 are also counted using the cube deletion process (Figure 6).

The next step is to check that all the empty boxes have been deleted using a wire frame model (Figure 7).

In this stage, it will be clear that the boxes starts to form the shape of the building – Figure 8, which relates the visual assessment process to the visual design process.

Iteration 3

In the third iteration, the box size is 1 meter, see figure 9. Moreover, by applying the cube deletion process, the resulting form is much more similar to the image of the building, and if the iteration continued to the minimum box size possible, the boxes formation will look exactly like a pixilated version of the original form, or even the result will be the form itself. The problem in a manual application is that 1 meter or 0.5 meter is the minimum box size to be workable; the cube deletion process will become impossible in a manual application if the box size is less than 0.5 meters for small buildings, or 1 meter for medium size ones like the Robie house.

The remaining boxes are then counted; the computer will give the result (N), which should be used in the box counting equation. Table 1 illustrates the results for each iteration.

Applying the equation:

$$D_b = \frac{\log (N(s2)) - \log (N(s1))}{\log (s2) - \log (s1)} \quad (7)$$

$D_{b\ (\text{box size } 6\text{-}3)}$ = 2.4-1.69 /1.3-1 = 2.36

$D_{b\ (\text{box size } 6\text{-}1)}$ = 3.65-1.69 / 1.77-1 = 2.54

$D_{b\ (\text{box size } 3\text{-}1)}$ = 3.65- 2.4 / 1.77-1.3 = 2.65 (most reasonable)

6. Validation concept

In a box counting method, the results has to be validated by confirming it in two levels of validity, starting by the mathematical validity, which requires that the box counting dimension has to be related to the topological dimension; if the box counting was performed two dimensionally, the result has to be between 1 and 2, and if it was performed three dimensionally, the results has to be between 2 and 3. If the results were found within those ranges, it proves that the process is mathematically valid, and the equation is applicable.

The second level of validity is to confirm that the resultant dimension represents the true value of visual complexity, and this can only be validated by comparing different samples. In a three dimensional box counting operation, the results of samples has to be validated in accordance with the results of a two dimensional box counting approach for the same samples, the levels of complexity should be in the same order in both two dimensional and three dimensional box counting operations; this will lead to an architectural problem, that some buildings are complex in some elevations and very simple in the other elevations, which means that a building might be more complex than its opponents in the three dimensional box counting, and yet it has the least visual complexity in two dimensional box counting,

Figure 10. Single façade complexity

because the chosen façade is the simplest side of the building; this problem might affect the validation process; therefore, to test the validity of the approach, the chosen samples has to be a single façade buildings, the kind of building that have only one complex elevation, and the rest of its sides are flat walls, (see the building in Figure 10), because in this case, the 2D complexity of the front façade will comparatively represent the same level the 3D complexity of the building.

For more validity check, a visual preference test to determine the experienced visual complexity order by a normal person might help confirming the box counting results, if the order of complexity calculated by box counting was the same order which has been chosen by normal random selected viewers, considering statistical validity of the visual preference test. If the approach of three dimensional box counting method was validated through such steps, then it can be computerized in further research, and then it can be used for all types of buildings to perform further test for its comparability and workability.

7. Conclusion

Discussing the fractal assessment tool (box counting) it has been found that most of its problems are caused by its two dimensional approach. The problems about 2D box counting were found to be:

1. The conceptual problem represented in neglecting the layout of the building and the reassesses and projections.
2. The perceptual problem represented in the scale range concept, and the box size perceptual meaning, and the reflection of human perception on the iterations of the approach.
3. The practical problem represented in the line width of samples, the amount of white space around the sample, and the level of details taken in the sample.
4. The mathematical problem represented in the amount of details within filled boxes as a statistical problem. And this problem has been mathematically neglected by the mathematicians who established the box counting equations.

7.1. The developed approach

3D box counting is considered a new approach of computational assessment, based on fractal geometry. Which delivered mathematically and conceptually valid results in the application on Robie house, using two major approach parameters: considering the building as solid volume, and using box elimination approach manually (cube deletion process). Figure 11 illustrates the parameters of the **3D box counting approach**.

The next step should be to apply this approach on a real case, handling an existing built form on multiple scales. Then to validate the results mathematically and geometrically through statistical analysis, then conceptually and architecturally through visual preference. This will also help determining the correct interpretation of results, and to assure that fractal dimension represents visual complexity.

8. Further research

A further development can be performed to this approach, by involving line segmentation approach to develop box counting method, using a wire frame model, based on the concepts of design by lines (Ando, N. 2007). Such development will increase the accuracy of 3D box counting on the level of lines representation, and will reconnect visual assessment process with visual design process. Line count and statistical assessment

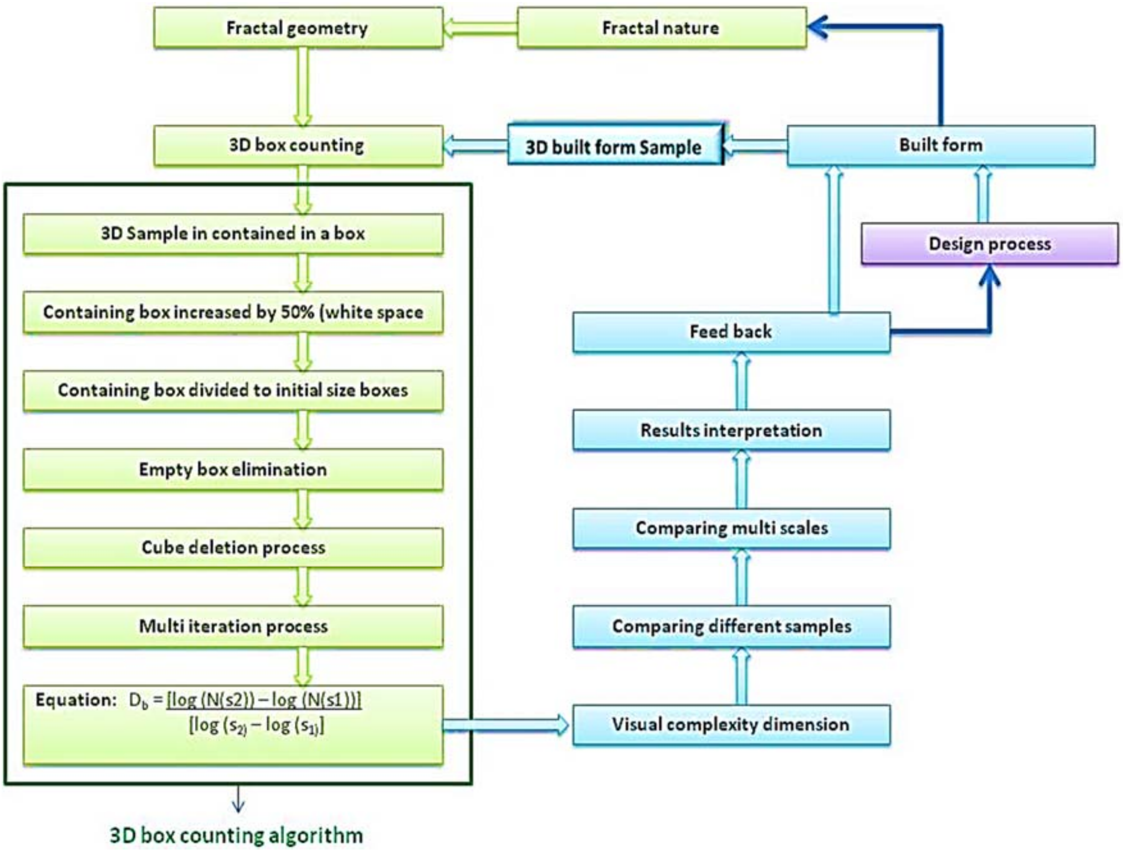


Figure 11. The developed approach

can be involved in such an approach. Also the use of visual data collection technologies such as laser scanning and edge detection algorithms will increase the workability of the approach.

8.1. Computerisation of the approach

The most important step in further development is computer application; by programming an algorithm that can perform 3D box counting as developed in this research and 3D line segmentation. This computerized algorithm can evolve this computational approach to a practical tool.

8.2. Making the approach usable and available

Computerization of this algorithmic approach with the use of data collection technologies, and involving computer vision can evolve this approach to become usable in practice outside the laboratories. Such technology can even be commercially developed and produced in mass scale with the improvement of easy user interface, to make available and usable for everyone, such as architectural designers, academic institutions, municipalities and planning authorities, in addition to heritage preservers. The simplicity of visual assessment is very important in order to involve its application in all types of architectural activities, and building related tasks, including identity preserving and academic research. Such an approach can be developed to be used even to evaluate students' projects in the academic field, and to judge design competitions, easily and accurately.

8.3. Developing computer software for visual assessment

Computer software has to be developed to apply the generated visual assessment algorithm. This software has to perform visual assessment process as the following:

- Provide a 3D environment with the capability to convert different 3D file formats into suitable 3D code, and it should be compatible with (3Ds, DWG, DXF, ...etc) and all 3D formats that generated by CAD, BIM or 3D laser extraction.
- When the 3D file is successfully loaded into the software interface, a scaling tool must be available along with measuring tool, in order to perform scaling correction, and it has to similar

to AutoCAD tool (scale by reference) to achieve maximum scaling accuracy.

- The software must then be able to generate 3D boxes rendered as transparent in a controllable initial size, multiple iterated sizes with the ability to control the number of iterations, accurate overall size (50% larger than model size), and smallest box size to be editable. This should come with a feature to control the graph of box sizes through the iterations to manipulate the distribution of the relative box sizes in all iterations. It should also be able to determine (S) value which is the number of boxes in the X axis.
- The detection process of which boxes are occupied by the building should follow the cube deletion process in order to detect even the boxes contain inside the building enclosure, because the building model will mostly be hollow, and the findings of the research requires the building to be treated as solid. This means that only the cubes that don't encounter the external skin and they are outside the enclosure, are to be deleted. To achieve this, the software should apply an algorithm that detects the boxes encountered by the skin and the boxes contained by them. As an alternative, the algorithm should delete only visible boxes unless they are encountered with the model skin (this is the algorithm used in the manual application). All this process must be performed in visible pattern in scene without rendering to minimize display processor load.
- The software must then repeat the process in all iterations and count and record the number of boxes in each iteration, providing a table of values that can be exported via exile format. The software must be able to export 3D files for the resulting boxes shape of each iteration. This will allow manual follow up, and will help the research process by providing detailed data.
- The software must not delete the resulting boxes while following the iterations, it should rather perform each iteration in an auto-named layer; this should allow the software user to re display each integration of hide it using layer control tools.
- The equation of box counting should be applied through all iterations and between each two generating fractal dimension values, providing all the results in table and graph format. The graph format must contain iteration names as on axis and fractal dimension values on the other.

$$D_b = \frac{[\log (N(s2)) - \log (N(s1))]}{[\log (s2) - \log (s1)]}$$

- The software must be able to perform results check and choose a final result by finding the most frequent result. A statistical analysis tool must be available to re-evaluate the results.
- The software must be able to store the visual assessment procedure via special file format containing all the tables, graphs and 3D data. It should also be able to compare different file results and provide statistical reports and graphs based on multiple files.
- Such software should have a collaborative interface with BIM software in order to compare fractal dimension of new designs with existing built form. It will be even suitable to provide a version of the software via plug-in for BIM programs to allow checking fractal dimension during design.

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